

Warm-ups

① $f'(1) = 0$
 $f''(x) = x^2 e^{\cos(3x)}$

What can you say about $x=1$?

② Find $\int \frac{\sin(2x)}{1 + \cos^2(2x)} dx$

① Since $f'(1) = 0$, $x=1$ is a cr pt.

Since $f''(1) = 1^2 e^{\cos(3)} > 0$, graph is concave up.

✓ $\therefore x=1$ is a local min

② $\int \frac{\sin(2x)}{1 + \cos^2(2x)} dx$

let $u = \cos(2x)$

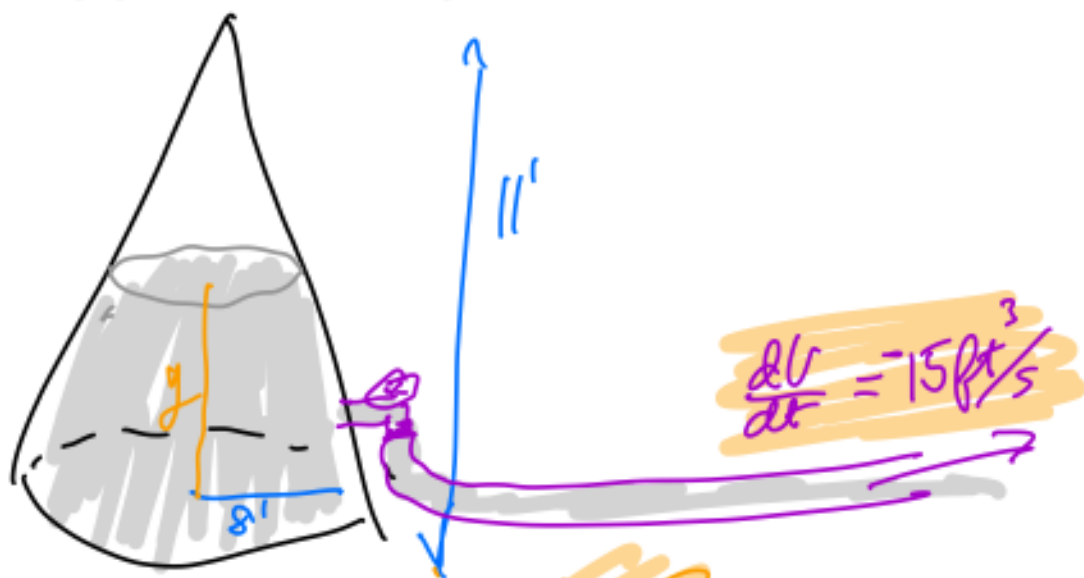
$du = -2\sin(2x) dx$

$-\frac{1}{2} du = \sin(2x) dx$

$= \int \frac{-\frac{1}{2} du}{1+u^2} = -\frac{1}{2} \int \frac{du}{1+u^2} = -\frac{1}{2} \arctan(u) + C$

$= -\frac{1}{2} \arctan(\cos(2x)) + C$

25. A cone-shaped container (of radius 8 feet, height 11 feet) contains molten lead. If the lead is pumped out of the container at a rate of 15 cubic feet per second, how fast (in feet per second) is the height changing when the container is $1/4$ full?



$\frac{1}{4}$ full

~~$y = \frac{11}{4}$~~

$\frac{dy}{dt} = ?$

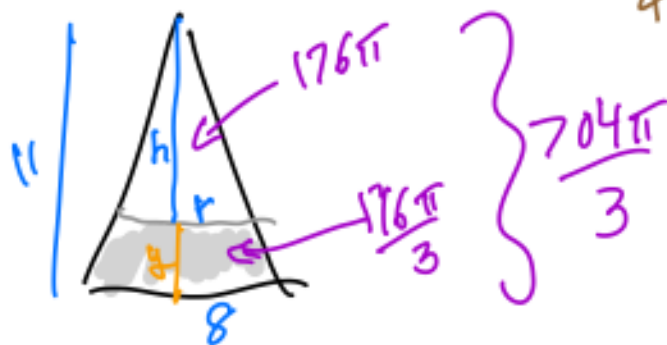
whole volume:

$$\frac{\pi r^2 y^2}{3} = \frac{\pi 8^2 \cdot 11}{3}$$

$$\text{Whole volume} = \boxed{\frac{704\pi}{3}}$$

$$\text{Current volume} = \frac{1}{4} \frac{704\pi}{3} = \frac{176\pi}{3}$$

$$\text{Air volume} = \frac{3}{4} \frac{704\pi}{3} = 176\pi$$



$$h = 11 - y$$

$$y = 11 - h$$

$$\text{Similar triangles: } \frac{8}{11} = \frac{r}{h}$$

$$\Rightarrow h = \frac{11}{8} r$$

$$\Rightarrow r = \frac{8}{11} h$$

$$\frac{1}{3} \pi r^2 h = 176\pi \Rightarrow \frac{1}{3} \pi \left(\frac{8}{11} h \right)^2 h = 176\pi$$

$$\frac{1}{3} \frac{8^2}{11^2} h^3 = 176$$

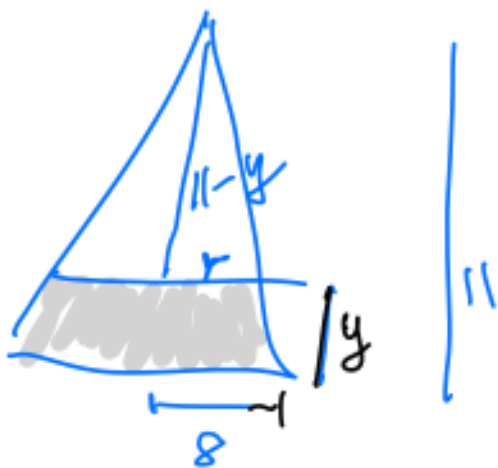
$$h^3 = \frac{176 \cdot 11^2 \cdot 3}{8^2}$$

$$h = \sqrt[3]{\frac{176 \cdot 11^2 \cdot 3}{8^2}} = 9.99416'$$

$$\approx 10$$

$$y = 11 - h = 1$$

we need an equation that
has y & V in it so we
can relate $\frac{dy}{dt}$ & $\frac{dV}{dt}$.



whole vol.

$$V = \frac{1}{3} \pi 8^2 \cdot 11$$

air volume

$$- \frac{1}{3} \pi r^2 (11-y)$$

simila

$$\frac{r}{11-y} = \frac{8}{11}$$

$$r = \frac{8}{11} (11-y)$$

$$V = \frac{1}{3} \pi 8^2 \cdot 11 - \frac{1}{3} \pi \left(\frac{8}{11} (11-y) \right)^2 \cdot (11-y)$$

$$V = \frac{1}{3} \pi 8^2 \cdot 11 - \frac{8^2 \pi}{3 \cdot 11^2} (11-y)^3$$

derivative

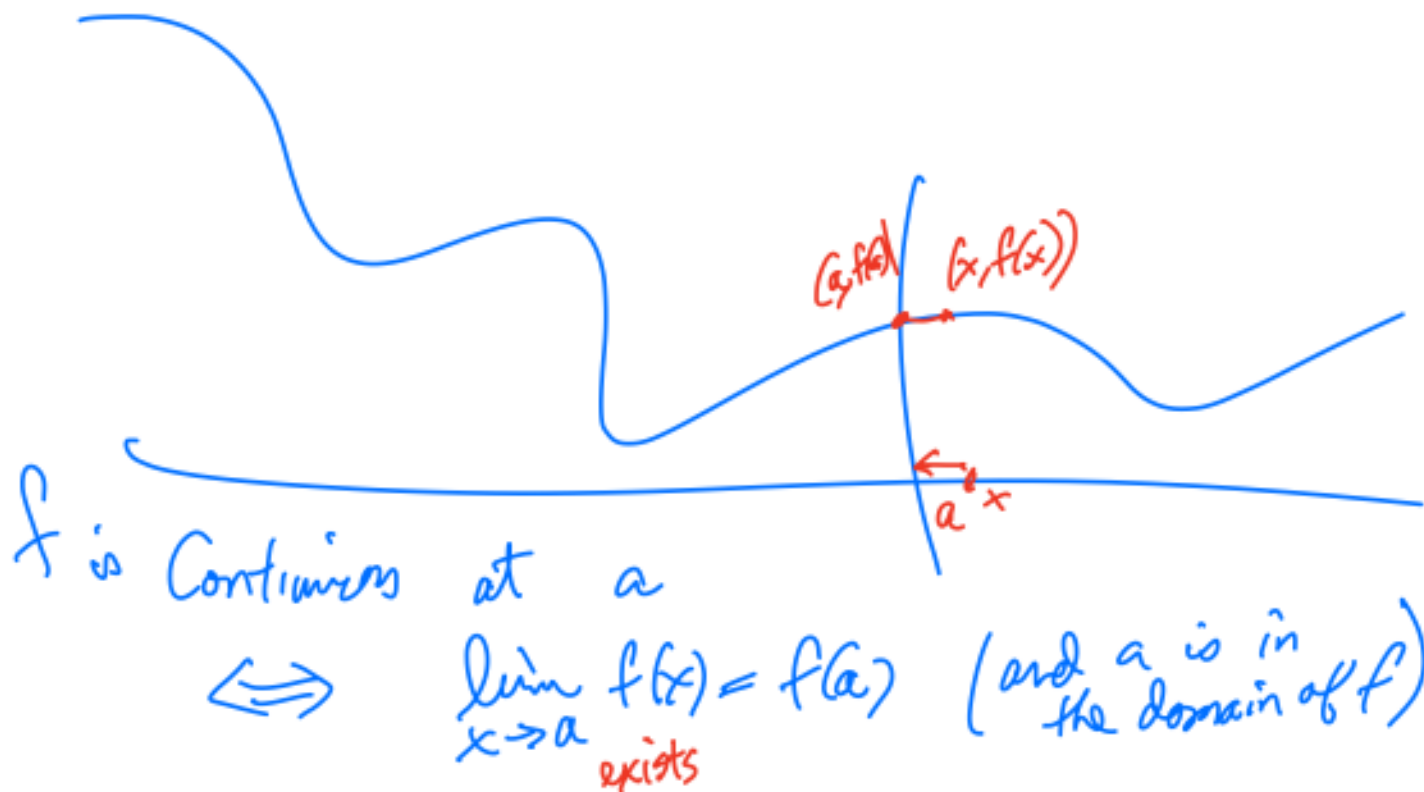
$$\frac{dV}{dt} = 0 - \frac{8^2 \pi}{3 \cdot 11^2} \cdot 3(11-y)^2 \cdot \left(\frac{dy}{dt}\right)$$

$$\Rightarrow \frac{dV}{dt} = \frac{8^2 \pi}{11^2} (11-y)^2 \frac{dy}{dt}$$

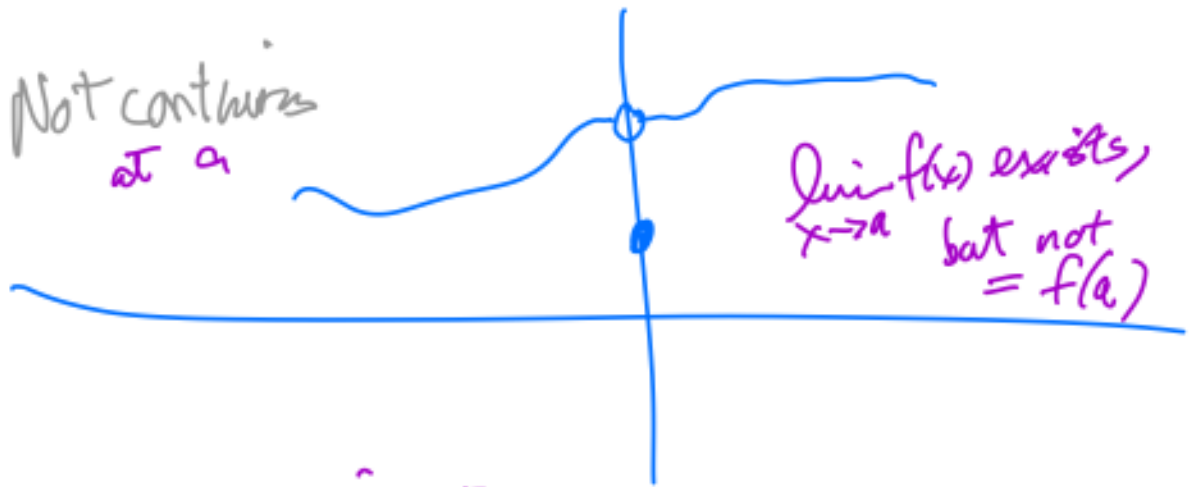
$$\Rightarrow -15 = \frac{8^2 \pi}{11^2} (11-10)^2 \frac{dy}{dt}$$

$$\Rightarrow \left(\frac{-15 \cdot 11^2}{8^2 \pi 10^2} \right) = \frac{dy}{dt} = \boxed{-0.09 \text{ ft/s}}$$

$$= \boxed{1.1 \text{ inch/s}}$$

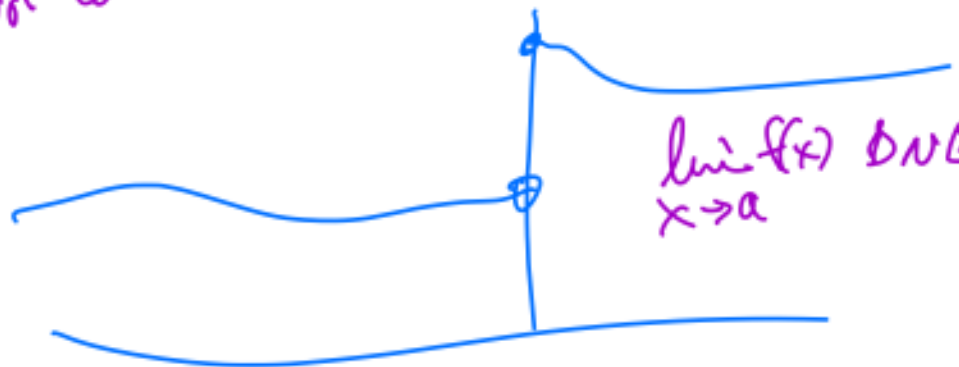


Not continuous
at a



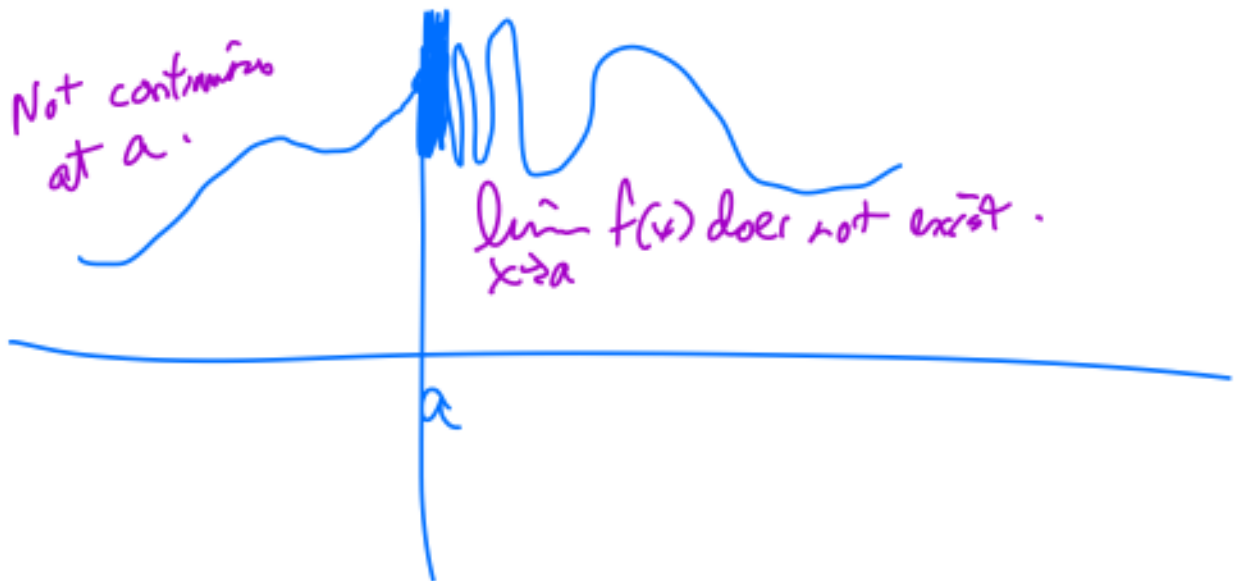
$\lim_{x \rightarrow a} f(x)$ exists,
but not
 $= f(a)$

Not continuous at a



$\lim_{x \rightarrow a} f(x)$ DNE

Not continuous
at a .



$\lim_{x \rightarrow a} f(x)$ does not exist.